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A Pricing Kernel Approach to Valuing Options on Interest Rate Futures

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Abstract

This paper builds on existing asset pricing models in an intertemporal CAPM framework to investigate the pricing of options on interest rate futures. It addresses the issues of selecting the preferred pricing kernel model by employing the second Hansen-Jagannathan distance (HJD) criterion. This criterion restricts the set of admissible models to those with a positive stochastic discount factor that ensures the model is arbitrage free. The results indicate that the 3-term polynomial pricing kernel with three non-wealth-related state variables, namely the real interest rate, maximum Sharpe ratio, and implied volatility, clearly dominates the other candidates. This pricing kernel is always strictly positive and everywhere monotonically decreasing in market returns in conformity with economic theory.

JEL code: C11, G12, G13

Keywords: Pricing Kernels, Simulation-based Bayesian Approach, LIBOR Futures Options

1 Introduction

This paper investigates the pricing of options written on 6-month LIBOR (London Interbank Offer Rate) futures in an intertemporal CAPM framework. LIBOR is the most commonly cited base rate in global money and debt markets including the Eurocurrency and Eurobond markets. It is the daily reference interest rate at which banks borrow unsecured funds from other banks in the London wholesale money market. The overnight LIBOR serves as the benchmark for short-term interbank loans that are a cornerstone of the money markets while the 3-month LIBOR is employed as the base rate for short and medium term corporate loans and for floating rate corporate bonds. Moreover, the 6-month LIBOR is widely used in the burgeoning over-the-counter derivatives markets such as interest rate and currency swaps. Hence the hedging of LIBOR exposure is a pervasive problem and the pricing of hedging instruments such as LIBOR options becomes an important issue.

There are two broad approaches in the literature to pricing options. One widely adopted approach involves specifying the dynamics of the underlying asset and solving for the closed-form solution. Examples include Bakshi, Cao, and Chen (1997), Bates (1996), Pan (2002) for stock index options and Hull and White (1990), Heath, Jarrow, and Morton (1992), and Singleton and Umantsev (2002) for interest rate options. Alternatively, a more general approach to evaluating the prices of risky assets, including derivatives, is to employ an asset pricing kernel, also known as a stochastic discount factor (SDF). The pricing kernel is a strictly positive random variable that succinctly summarizes investor risk and time preferences with

respect to financial assets. It is used to compute today's asset price by stochastically discounting, state by state, the corresponding payoffs at future dates (Harrison and Kreps (1979), Hansen and Jagannathan (1991, 1997), Bansal and Viswanathan (1993), and Chapman (1997)).

The paper makes two contribution to the literature. Firstly, our approach to option pricing builds on the work of Brennan, Wang, and Xia (2004) and Nielsen and Vassalou (2006) that establishes the important role of pricing kernels within the intertemporal CAPM framework of Merton (1973). Our study applies to options on LIBOR futures the parametric pricing kernel approach that has been successfully employed in pricing stock index options (Rosenberg and Engle (2002), Jones (2006), and Brennan, Liu, and Xia (2007)) and stock portfolios (Dittmar (2002) and Vanden (2004)).

Brennan, Wang, and Xia (2004) stress that the distinguishing characteristic of the ICAPM is that the state variables in the pricing kernel are not simply any factors correlated with returns as in arbitrage pricing theory. Instead the innovations in state variables are able to predict future asset returns. They postulate that the intercept and slope of the instantaneous capital market line are sufficient to describe the innovations in the investment opportunity set. This is their theoretical justification for restricting the number of priced state variables to two in an ICAPM framework. *This ICAPM with aggregate wealth, and real interest rate and maximum Sharpe ratio as the two non-wealth-related state variables is found to dominate the Fama-French three-factor model and the CAPM in pricing US equity portfolios (Brennan, Wang, and Xia (2004)).* This is the basis for our choice of the real in-

terest rate and maximum Sharpe ratio as candidate state variables. We require no assumptions about the dynamics of interest rates or their term structure.

In the options market, there is strong empirical evidence that market volatility is priced with a negative risk premium (Coval and Shumway (2001) and Bakshi and Kapadia (2003)). Based on these findings, we consider the LIBOR option implied volatility as a further non-wealth-related state variable for our pricing kernel. We use an exponential affine function with time-varying innovations to ensure that the pricing kernel is nonlinear in the three state variables and hence capable of pricing nonlinear payoffs (Chapman (1997) and Dittmar (2002)). We evaluate two functional forms for market returns, a nonlinear power function and a linear Chebyshev polynomial approximation. Both are popular choices in the equity and option pricing literature (Brennan (1979), Chapman (1997), Rosenberg and Engle (2002), and Brennan et al. (2007)). The use of these functional forms ensures comparability between our results and those of previous studies.

While Brennan et al. (2004) sort their equity portfolios on the basis of size and book-to-market ratio, we use option moneyness to sort our option portfolios. Using monthly moneyness-based portfolio returns on LIBOR options from January 2000 to February 2008, our results indicate that the coefficients of the real interest rate, the maximum Sharpe ratio and implied volatility are all statistically significant regardless of the functional form. It is in line with the extant literature such as Collin-Dufresne and Goldstein (2002), Bakshi and Kapadia (2003), Brennan, Wang, and Xia (2004), Nielsen and Vassalou (2006), Li and Zhao (2006), Bollerslev and Zhou (2009) and Carr and Wu (2009) for the role of these state variables. This is also

consistent with the findings in Brennan et al. (2007) who use the same functional forms for index options in the US market and find that all candidate state variables are priced. The implication is that options on LIBOR futures can be priced by means of a similar set of state variables as that for index options.¹ *This is also consistent with the theoretical derivation which indicates that futures prices behave like stocks with dividend payments in a risk-neutral world, and that the behaviour of put-call parity for options written on futures contracts and those written on stock indices is the same (Hull (2008)).*

The second contribution is that methodologically our study is one of the first papers that apply the second Hansen-Jagannathan distance (HJD) in evaluating candidate pricing kernels. There are two motivations for this. On the one hand, the usual GMM estimation produces problematic pricing kernels that either fail statistical robustness tests or are inconsistent with economic theory by producing negative or hump-shaped pricing kernels. On the other hand, the first HJD, which measures pricing errors over the test portfolios and has been widely applied in the literature (Jagannathan and Wang (1996), Buraschi and Jackwerth (1999), Dittmar (2002), Lettau and Ludvigson (2001), among others), shares some of the problems inherent in the GMM approach. By contrast, the second HJD measure restricts our focus to the family of strictly positive pricing kernels only. This positivity constraint guarantees that the pricing kernels are arbitrage-free, an essential requirement for correctly pricing contingent claims.

¹One minor difference is that they employ implied market volatility whereas we use implied LIBOR volatility as our third state variable.

However, the second HJD has rarely been applied in the literature mainly due to the difficulty in deriving a reliable posterior distribution for the test statistic. Notable exceptions include Hansen, Heaton, and Luttmer (1995), Wang and Zhang (2012), and Li, Xu, and Zhang (2010). Hansen et al. (1995) develop an asymptotic distribution for the sample estimate of the second HJD under the null hypothesis that a given stochastic discount factor (SDF) model is misspecified. However, the asymptotic theory no longer holds under the null hypothesis of a correctly specified model or when the second HJD is zero. Wang and Zhang (2012) propose a simulation-based Bayesian approach that facilitates statistical inference for the second HJD in finite samples. Bayesian methods provide us with the full posterior density of the model parameters and hence the full posterior of the second HJ distance, resulting in inference that takes into account parameter uncertainty and is valid in finite samples (Koop (2003)). More recently, Li, Xu, and Zhang (2010) adopt the second HJD as the yardstick for comparing alternative asset pricing models within a conventional econometric framework.

We follow Wang and Zhang (2012) and adopt the Bayesian econometrics approach to provide a robustness test for the second HJD in estimating the pricing kernels. This is the first study that implements this approach for interest rate options. Our results indicate that the linear Chebyshev polynomial pricing kernels outperform the non-linear power function models in producing smaller pricing errors. In addition, unlike the linear pricing kernels obtained from the GMM, those obtained via the second HJD conform neatly with economic theory by being strictly positive and decreasing in market returns. These findings underline the inherent advantage of

the second HJD over competing statistical measures in evaluating pricing kernels for derivatives. The 3-term generalized Chebyshev polynomial model with three non-wealth-related state variables has the smallest second HJD and hence emerges as the preferred functional form for pricing options on LIBOR futures.

The rest of the paper proceeds as follows. Section 2 discusses the parametric functional forms of the pricing kernel, the state variables, and the second HJ distance. Section 3 describes data and analyzes the empirical results. Finally, Section 4 concludes.

2 Methodology

2.1 The state variables

The importance of including non-wealth-related state variables in pricing kernels has been widely stressed in the literature. The main reason is that such variables enhance the ability of pricing models in capturing time-varying investment opportunities (Garcia, Luger, and Renault (2003), Vanden (2004), Santa-Clara and Yan (2010), among others). Nielsen and Vassalou (2006) postulate that the intercept and slope of the instantaneous capital market line are sufficient to describe the innovations in the investment opportunity set in the context of portfolio hedging. Supportive empirical evidence is given in Brennan, Wang, and Xia (2004).

We follow the above authors in choosing the real interest rate r , and the maximum Sharpe ratio η as state variables. There is also strong empirical evidence that market

volatility is priced in the options market. Thus we include implied volatility σ as a potential third state variables for pricing LIBOR futures options. This particular set of state variables $X \equiv (r, \eta, \sigma)$ facilitates comparison with the results in Brennan et al. (2007), who adopt the same set of state variables for pricing stock index options in the US and the UK markets. Hence, this study may be able to shed light on whether the state variables are distinct for pricing options on different financial assets.

We assume that the real interest rate and the maximum Sharpe ratio follow correlated Ornstein-Uhlenbeck processes.² With further assumptions, the model can be adapted to the pricing of default-free nominal government bonds.³ The model parameters and the time series of these two state variables can be estimated from panel data on UK nominal zero-coupon government bond yields and inflation via a Kalman filter. *Following Brennan, Wang, and Xia (2004) and Brennan and Xia (2006), we assume the real interest rate and maximum Sharpe ratio follow correlated Ornstein-Uhlenbeck processes and together they define the stochastic discount factor,*

$$\frac{dm}{m} = -r dt - \eta dz_m \quad (1)$$

$$dr = \kappa_r(\bar{r} - r)dt + \sigma_r dz_r \quad (2)$$

$$d\eta = \kappa_\eta(\bar{\eta} - \eta)dt + \sigma_\eta dz_\eta \quad (3)$$

where κ_r and κ_η are the speed of mean reversion for the real interest rate and the

²It is reasonable to assume that the state variables follow the Ornstein-Uhlenbeck process, a mean-reverting stochastic process. In our framework, the real interest rate is stochastic and the risk premia are a part of the Shape ratio (Brennan et al. 2004). Hence, the risk premia are assumed to follow a simple diffusion process with a mean-reverting drift (see also Kim and Omberg (1996)).

³See Brennan et al. (2004) for details.

Sharpe ratio, \bar{r} and $\bar{\eta}$ are the long-term mean, σ_r and σ_η are the volatility of the two processes, respectively, and the correlation between them is $\rho_{r\eta}$. More specifically, $\rho_{r\eta}$ is the correlation coefficient between the Wiener processes for the real interest rate and the maximum Sharpe ratio. Implied volatilities are inferred from individual options and averaged to obtain portfolio volatility.

2.2 The pricing kernels

The pricing kernel approach has been widely employed in the asset pricing literature (Breedon (1979), Cochrane (1996), Abel (1990), among others). Cochrane (2005) argues that the projected pricing kernels onto the asset return space have the same pricing implications as the true pricing kernels. As a result, the portfolio choice problem for any investor can be solved by the Euler equation

$$E \left[m_{t+1} \tilde{R}_{i,t+1} | \Omega_t \right] = 1 \quad (4)$$

where m_{t+1} is the pricing kernel, a function of state variable set X ; $\tilde{R}_{i,t+1}$ is the gross return on an asset or portfolio i at time $t + 1$; and Ω_t is the information available at time t . The pricing kernel is also known as the stochastic discount factor since it varies over time and across states and can be applied to compute the expected discounted return that should always be equal to unity.

Motivated by Rosenberg and Engle (2002), two basic forms of the pricing kernel are implemented. These are a nonlinear power function and a linear Chebyshev polynomial expansion in aggregate wealth growth, both of which are augmented by an exponential affine function of innovations in the state variables. The use of these

functional forms in the pricing kernels provides the basis for a comparison between linear and nonlinear forms of pricing kernels.

Our choice of candidate functional forms is based on firm theoretical underpinning. Under the assumptions of constant relative risk aversion (RRA) for economic agents and a bivariate normal distribution for asset returns and aggregate wealth growth, Rubinstein (1976) and Brennan (1979) demonstrate that the Black-Scholes option pricing model implies, in a discrete time setting, a power function: $m^* = k(\tilde{R}^{-\gamma})/R_f$. Here R_f is the riskfree interest rate, k is a constant, and γ is the RRA parameter. In continuous time, Bick (1987) uses the same projected pricing kernel but with a continuously compounding interest rate in the Black-Scholes framework. More generally, Dybvig (1981) indicates that the projected pricing kernel implied by the Black-Scholes model is equivalent to a power function of the gross return on aggregate wealth discounted by the continuously compounded interest rate $m^* = k(R_W^{-\gamma})e^{-r}$ where $k = (E[R_W^{-\gamma}])^{-1}$.

In light of the theoretical link between the Black-Scholes model and the pricing kernel approach, we first assume that the pricing kernel is expressed as a power function of aggregate wealth returns, R_w , augmented by an exponential affine function of the innovations in the state variables as follows,⁴

$$m_{t+1} = \beta(R_{w,t+1})^{-\gamma} \exp^{b_1\Delta r_{t+1}+b_2\Delta\eta_{t+1}+b_3\Delta\sigma_{t+1}} \quad (5)$$

⁴ *The exponential affine functional form is adopted to make the pricing kernels nonlinear in order to capture nonlinear option payoffs. Other methods of making pricing kernels nonlinear include having higher moments of returns in the pricing kernels (Dittmar (2002)) or adding cross terms between returns and conditioning variables (Wang and Zhang (2012)).*

where β , b_1 , b_2 , and b_3 are constants and γ is the RRA parameter. This iso-elastic function captures the decreasing marginal utility of wealth. *However, its main drawback is that it has weak statistical robustness, as pointed out by Hansen and Singleton (1982).*

The second functional form of the pricing kernel is a linear Chebyshev polynomial in aggregate market returns. Chapman (1997) discusses the benefit of approximating pricing kernels by means of polynomials. Such an approach combines linearity in the functional form with nonlinearity in the state variables. Hence it is capable of pricing nonlinear payoffs while retaining linear interpretation. Our second candidate pricing kernel is expressed as the sum of Chebyshev polynomials augmented by an exponential affine function of the innovations in the state variable as follows,

$$m_{t+1} = \wp^n(R_{w,t+1}) \exp^{b_1 \Delta r_{t+1} + b_2 \Delta \eta_{t+1} + b_3 \Delta \sigma_{t+1}} \quad (6)$$

where $\wp^n(R_{w,t+1})$ consists of an n -term Chebyshev polynomial. We follow Brennan et al. (2007) and Chapman (1997) and use both 3- and 4-term polynomial approximations.

2.3 The second HJD and Markov Chain Monte Carlo (MCMC)

Bayesian inference

Following Wang and Zhang (2012), we estimate the pricing kernel parameters, θ , by minimizing the second HJD $Dist_{HJ2}$.⁵ We obtain the posterior mean of $Dist_{HJ2}^A / Dist_{HJ2}^B$ for comparing the second HJ distances of candidate pricing kernels

⁵The interested reader is referred to Wang and Zhang (2012) for more details.

from a Markov Chain Monte Carlo (MCMC) simulation-based Bayesian approach. Let z_t be a matrix of size $t \times N + l + k$ composed of N asset returns r_t , l state variables s_t , and k factors f_t which include all other information like polynomial terms, thus $z_t = (r'_t, f'_t, s'_t)'$. According to Hansen and Jagannathan (1997), the second HJD is defined as

$$\text{Dist}_{HJ2} = \left\{ \max_{\lambda \in R^N} E[m_t^2 - ([m_t - \lambda'x_t]^+)^2 - 2\lambda'q_t] \right\}^{1/2} \quad (7)$$

where m_t is the candidate pricing kernel, \mathbf{R}^N is the space of $N \times 1$ real vectors, x_t stands for asset payoffs, q_t denotes asset prices, $[m_t - \lambda'x_t]^+$ produces non-negative values and λ' is a vector of n parameters which are estimated to solve the optimization problem of the second HJ distance.

For the data-generating process in the MCMC simulation, z_t is assumed to follow a VAR, hence $z_t = C + Az_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0_m, \Omega)$. Under the assumption of independent non-informative prior distributions for the unknown parameters, z_0 , B and Ω , we have $p(\Phi) = p(z_0)p(B)p(\Omega)$, where $\Phi = (z'_0, \text{vec}(B)', \text{vech}(\Omega)')'$, $p(z_0) \propto \text{constant}$, $p(B) \propto \text{constant}$, $p(\Omega) \propto |\Omega|^{-(m+1)/2}$, $m = N + k + l$, and B is the matrix of parameters including C and A in the VAR system. Note that vec denotes the conversion from a matrix to a vector when all the columns in a matrix are stacked, while $\text{vech}(\Omega)$ is the vector converting the upper triangle of matrix Ω .

The MCMC simulation method is applied to tackle the difficulty in deriving the posterior distribution of the unknown parameter set Φ . The intuition is that z_t is assumed to follow a general stochastic process which is determined by the unknown parameters Φ and the conditional probability of z on unknown parameters $p(z|\Phi)$.

With the assistance of the Markov Chain Monte Carlo simulations, we can obtain

a reliable posterior distribution of the parameters ϕ and subsequently a reliable posterior distribution of the second HJ distance. We carry out $S = 10,000$ simulations and discard the first $S_0 = 1000$ simulations to approximate the posterior distribution for the second HJ distance. More specifically, we implement the following procedures to compare the second HJD of different candidate pricing kernels.

In the first stage, we choose an arbitrary $z_0^{(0)}$, and perform the simulations ($j = 1, \dots, S$):

1. Obtain the j th sample of unknown parameters from their conditional posterior distributions

- Draw $\Omega^{(j)}$ from $IW\left(T\hat{\Omega}(z_0^{(j-1)}), T-1, m\right)$, where IW is the inverted Wishart distribution;⁶
- Draw $\text{vec}(B^{(j)})$ from the truncated normal distribution

$$N\left(\text{vec}(\hat{B}(z_0^{(j-1)})), \Omega^{(j)} \otimes [X(z_0^{(j-1)})'X(z_0^{(j-1)})]^{-1}\right).$$

We limit the norm of the eigenvalues of parameter matrix A to be less than unity to ensure that the VAR is stationary.

- Draw $z_0^{(j)}$ from $N\left([A^{(j)}]^{-1}(z_1 - C^{(j)}), [A^{(j)}]^{-1}\Omega^{(j)}[A^{(j)'}]^{-1}\right)$ where

$$\hat{\Omega}(z_0) = \frac{1}{T}[Z - X(z_0)\hat{B}(z_0)]'[Z - X(z_0)\hat{B}(z_0)]$$

⁶ The Wishart distribution is the conjugate prior to the inverse covariance matrix of a multivariate normal random vector. It is employed as the distribution of the sample covariance matrix from a multivariate normal distribution. The inverted Wishart distribution is an appropriate choice for the first step of simulations given our assumption that z_t follows a VAR and its disturbance follows a normal distribution with zero means and a covariance matrix of Φ .

$$X(z_0) = \left((1, z'_0)', (1, z'_1)', \dots, (1, z'_{T-1})' \right)'$$

$$\hat{B}(z_0) = [X(z_0)'X(z_0)]^{-1}X(z_0)'Z$$

$$Z = (z_1, \dots, z_T)'.$$

2. Obtain the j th sample with unconditional mean $\tilde{\mu}(\nu_t)$ and variance $\tilde{\Sigma}(\nu_t)$

$$\tilde{\mu}(\nu_t)^{(j)} = \tilde{C}^{(j)} + \tilde{A}^{(j)}\mu(z_t)^{(j)}$$

$$\tilde{\Sigma}(\nu_t)^{(j)} = \tilde{A}^{(j)}\Sigma^{(j)}\tilde{A}^{(j)'} + D\Omega^{(j)}D'$$

where

$$\mu(z_t)^{(j)} = \left(I_m - A^{(j)} \right)^{-1} C^{(j)}$$

$$vec(\Sigma(z_t)^{(j)}) = \left(I_{m^2} - A^{(j)} \otimes A^{(j)} \right)^{-1} vec(\Omega^{(j)}).$$

We assume that z_t follows a vector autoregressive (VAR) process, and the sample v_t is equal to $\tilde{C} + \tilde{A}z_{t-1} + D\varepsilon_t$. Therefore, D is the vector of parameters on the noise term and \otimes is the Kronecker product indicating element by element multiplication.

3. Compute the value of the second HJD for the j th sample.

In the second stage, we compute the posterior mean of second HJD of candidate pricing kernels and the posterior mean of $Dist_{HJ2}^A/Dist_{HJ2}^B$ for comparing the second HJD. Here, $Dist_{HJ2}^A$ is the second HJD of the candidate pricing kernels, while $Dist_{HJ2}^B$ is the one with smallest second HJD over the tested portfolios. The candidate pricing kernel with the smallest second HJD is chosen.

3 Data and empirical analysis

3.1 Data

We use settlement prices for 6-month LIBOR futures options traded in the London International Financial Futures and Options Exchange (LIFFE) from January 2000 to February 2008. We exclude options whose prices are below 5 pence or have less than 14 day to maturity to avoid potential stale prices and microstructural issues. We calculate monthly returns for all the options as long as they are traded for two consecutive months. We group option returns into five put and call portfolios according to their moneyness, defined as the difference between the underlying asset price and the strike price of the option and then divided by the strike price. The moneyness classes are chosen so that numbers of options are approximately evenly spread across the portfolios.

Summary statistics for the option return portfolios are reported in Table 1.

[Insert Table 1 around here]

All the call option portfolios have positive 1-month returns. Put option returns tend to be negative except in one case but the returns are less negative for deep in-the-money (ITM) put portfolios. According to the Jarque-Bera test, the null hypothesis of a normal distribution is rejected at the 5% level for all the portfolios except the most ITM call portfolio.

For options written on LIBOR futures, the underlying futures level is calculated as 1-LIBOR rate, so higher interest rate risk corresponds to lower underlying asset

price in a manner similar to the index options. Hence it is not surprising to observe similar patterns in the portfolio option returns in Table 1, as out-of-the-money (OTM) put index options are often overpriced with low expected returns as a precaution against extreme events like market crashes (Jackwerth (2000)).

3.2 Empirical results

In the GMM estimation, we employ the following set of instrumental variables: a constant, the real interest rate, the maximum Sharpe ratio, and the implied volatility. Table 2 provides summary statistics for bond yields, the state variables and their innovations.

[Insert Table 2 around here]

The data for inferring the state variables consist of the UK government bond yields of different maturities from January 1996 to February 2008 that we obtain from the Bank of England website. We use the longest time series of data available from this source to have more reliable estimation for the state variables.

In Panel A, we can see a slow increase in monthly yields with increasing maturity and a rather stable and small standard deviation. Using these bond yields of different maturities and the UK inflation rate, also taken from Bank of England website, the two state variables, r and η , are obtained via a Kalman filter. The implied volatility, σ , is obtained as the average of the implied volatility of individual options in the portfolio.

In Panel B, we tabulate summary statistics for the state variables and their innovations. We notice that the average return from the nominal interest rate, taken as the midpoint between LIBOR and LIBID (London interbank bid rate), is very close to the return for holding the market portfolio. This is due to a sharp correction in the market in the late 1990s and early 2000s. The returns to aggregate wealth, r_W , are proxied by the FTSE 100 index returns, which have a positive return with a large standard deviation of 0.48.

Our main empirical results are summarized in Tables 3 and 4. Table 3 presents the results from the GMM estimation.

[Insert Table 3 around here]

Panel A presents the parameter values for the iso-elastic power pricing kernel. We first include all three candidate state variables in the pricing kernel. The relative risk aversion parameter γ is 1.17, which is much smaller than the estimate of 4.05 in Bliss and Panigirtzoglou (2004) for 4-week UK stock index options with a power utility function. The coefficient for the real interest rate is 14.20 and significant. The positive coefficient is consistent with previous evidence of a negative risk premium associated with interest rate risk in Brennan et al. (2004) and Brennan et al. (2007). The coefficient for the maximum Sharpe ratio is -0.58 and also significant, in line with the findings in Nielsen and Vassalou (2006) and Brennan et al. (2004). This implies a positive risk premium for this state variable. The coefficient for volatility risk is 12.94 and significantly, consistent with findings in Collin-Dufresne and Goldstein (2002), Bakshi and Kapadia (2003) and Li and Zhao (2006) for pricing stock index and interest rate options and Bollerslev and Zhou (2009) and Carr and Wu (2009)

for stock index. In addition, for iso-elastic power kernels the test for over-identifying restrictions is clearly rejected regardless of the state variables. This points to a lack of statistical robustness for this functional form (Hansen and Singleton (1982)).

Interestingly, for the Chebyshev polynomials, the value for the over-identifying test J_T is greatly reduced and now the over-identifying restrictions are all accepted except one. This implies improved overall robustness. The coefficient for volatility is still significantly positive for the 3-term polynomial pricing kernels reported in Panel B and but not for 4-term ones in panel C. However, the over-identifying test of 3-term polynomial pricing kernels with three state variables is accepted, implying that the implied volatility should be included in the pricing kernel. The rejection of the over-identifying restrictions for the 4-term polynomial with three state variables could be due to the over-specification of the pricing kernel consisting of a large number of factors.

In addition, the 4-term polynomial with two state variables, the real interest rate and maximum Sharpe ratio,⁷ and the 3-term polynomial with all three state variables have the lowest value for the J_T test in Panels C and B, respectively. This indicates that all the three state variables are essential for pricing interest rate futures options.

The market-related component of the pricing kernels are shown in Figure 1.

[Insert Figure 1 around here]

⁷As the real interest rate and maximum Sharpe ratio are theoretically motivated together in Brennan et al. (2004) and Nielsen et al. (2006), we pair them together in the empirical tests.

We observe a high degree of variation in the scale and shape of the pricing kernels. Contrary to the literature, large sections of the polynomial pricing kernels are negative. There is a clear hump in the 3-term polynomials, while the 4-term polynomials exhibit an N-shape over the market returns. They indicate that investors are risk seeking in the positive slope regions and will pay to acquire fair gambles in wealth. This is not only contrary to economic theory but also counter-intuitive. More importantly, the negative pricing kernels reflect the existence of arbitrage opportunities, contrary to the fundamental assumptions of asset pricing theory.

Table 4 summarizes the empirical results when the coefficients are obtained by minimizing the second HJ distance.

[Insert Table 4 around here]

Panel A presents the parameter values for the iso-elastic power pricing kernel. We first include all three state variables in the pricing kernel. The coefficients for the real interest rate, maximum Sharpe ratio, and volatility are 21.03, -1.05, and 11.36, respectively, similar in magnitude to the values in Table 3. Using MCMC simulation-based Bayesian approach, the mean second HJD is 0.35 with three state variables in the pricing kernel. This is the lowest among the nonlinear iso-elastic pricing kernels with different state variables. This is additional evidence that all three state variables should be included in the pricing kernel.

We also test the pricing kernel with a smaller set of state variables with the real interest rate and maximum Sharpe ratio only. We notice that the parameter for relative risk aversion has a similar value, and the coefficients for the real interest

rate and maximum Sharpe ratio have the same sign and similar magnitude as in the previous specifications. When we remove the implied volatility and maximum Sharpe ratio from the pricing kernel and leave only the real interest rate, the mean second HJ measure increases from 0.35 to 0.39 while the coefficient of risk aversion becomes negative when there is no non-wealth-related state variable. These further emphasize the importance of incorporating non-wealth-related state variables in asset pricing kernels. The results also demonstrate that interest rate options can be priced by a similar set of state variables as for stock index options in previous studies. *Again, this is consistent with the extant literature that in a risk-neutral world, prices of stocks with dividends behave like futures prices.* The only difference is that the implied volatility of the LIBOR futures option is used in our case and that of the stock index option in the other studies.

In Panels B and C, we tabulate the parameter estimates for the 3- and 4-term polynomial pricing kernels, respectively. There is reasonably good consistency of the sign and magnitude of the parameter estimates between these two panels and also with Panel A. Specifically, in Panel B with third degree polynomial expansions, the mean second HJD measure drops from 0.12 to 0.08 when volatility is incorporated in the pricing kernel. Compared with the 3-term polynomial with real interest rate only, the second HJD is also reduced dramatically when the maximum Sharpe ratio is considered. These again confirm the necessary inclusion of these three state variables. In Panel C when there are four polynomial terms, although the pricing kernels could be more flexible with an additional term, the second HJD is similar or even slightly larger than that of the three-term polynomials. Similar to previous

results, the pricing kernel with all three state variables has the smallest mean second HJD.

In the last column of Table 4, we report the posterior mean of the ratio of second HJD between two candidate pricing kernels $Dist_{HJ2}^A/Dist_{HJ2}^B$ using MCMC simulation-based Bayesian approach. The second HJD of the 3-term polynomial approximation with all three state variables is employed as $Dist_{HJ2}^B$ as its mean is the lowest among all specifications. This implies that this pricing kernel is potentially the preferred one among all candidates. Therefore, the value in the last column is the posterior mean of the ratio between the candidate pricing kernels and the 3-term polynomial with three state variables.

As we expected, results show that the 3-term polynomial with three state variables is the preferred pricing kernel as the posterior mean of the ratio is the smallest. Similar to the rankings of the second HJD, the posterior means indicate that the 3-term polynomial outperforms the 4-term polynomial as well as the iso-elastic power function. Finally, consistent with the GMM estimates, the iso-elastic power function with three state variables has the smallest posterior mean among its family candidates.

The market-related component of the above pricing kernels is plotted in Figure 2.

[Insert Figure 2 around here]

All the pricing kernels are strictly positive thus offering no arbitrage opportunity. They are predominantly monotonically downward sloping,⁸ conforming to economic theories predicting a risk averse representative agent with diminishing marginal utility (Rubinstein (1976) and Lucas (1978)). *The pricing kernels depicted in Figure 2 are in contrast to the empirical pricing kernels recovered from the US and UK stock index options in Brown and Jackwerth (2004), Liu et al. (2009), and Rosenberg and Engle (2002). Although they adopt different methodologies over different sample periods, these papers all report hump-shaped pricing kernels. Our results show that utilizing information contained in non-wealth-related state variables and employing an econometric methodology that specifically addresses the issue of non-negativity in the SDF, we can produce empirical results that comply with theoretical predictions.*

Summarizing, for our sample period the 3-term polynomial approximation with three non-wealth-related state variables - the real interest rate, the maximum Sharpe ratio and the implied volatility - emerges as our preferred pricing kernel. After employing the second HJD as the objective function, even the linear pricing kernels are arbitrage free and monotonic. Therefore, the second HJD not only provides a robust criterion for testing the performance of candidate pricing kernels over contingent claims but also produces pricing kernels that are consistent with economic theory. Finally, our results indicate that the state variables for interest rate futures options are very similar to those for pricing stock index options in Brennan et al. (2007).

⁸For the 3-term polynomials with one and two state variables, there is a very small section of pricing kernels with a marginally positive slope when the market return is low.

3.3 Robustness test

Our full sample includes the onset of 2007-08 banking crisis. Thus we test whether the results are robust when we remove the banking crisis from the sample. Table 5 presents the empirical results from the GMM estimation using data up to December 2006.

[Insert Table 5 around here]

Panel A reports parameter values for the iso-elastic power pricing kernel when all three candidate state variables are included in the pricing kernel. The results are qualitatively similar to those for the full sample period except that now the coefficient of volatility risk is insignificant. Thus volatility risk is not priced for options prior to the onset of the banking crisis. Likewise, the coefficient for implied volatility is invariably insignificant for the polynomial pricing kernels reported in Panels B and C. The polynomial pricing kernel with two state variables, the real interest rate and maximum Sharpe ratio, now has the lowest value for the J_T test.

Table 6 summarizes the empirical results when coefficients are obtained by minimizing the second HJD using the pre-crisis observations only.

[Insert Table 6 around here]

Panel A summarizes parameter values for the iso-elastic power pricing kernel. Similar to the results for the whole sample, the average value of the second HJD is larger for iso-elastic power pricing kernel than that for polynomial approximations. The 3-term polynomial pricing kernel with two state variables in Panel B has the lowest average second HJD. Consistent with the GMM results in Table 5, only the

real interest rate and maximum Sharpe ratio are included in the pricing kernel and implied volatility is not priced. The plots of the market-related component of the polynomial pricing kernels estimated by second HJD are monotonically downward sloping, in conformity with economic theory.

We have undertaken two more robustness tests. In the first test, we relax the VAR(1) assumption in Section 2.3 and assume that the variables follow a VAR(2) in the data-generating process. In the second robustness test, we assume that the sample in the data-generating process follows a Student t distribution rather than a normal distribution. We then re-compute the corresponding second HJ distance. The new empirical results are qualitatively the same as the previous results, namely, the 3-term generalized Chebyshev polynomial model with three non-wealth-related state variables has the smallest second HJ distance and hence remains our preferred functional form for the whole sample. In addition, for the pre-crisis sample, volatility is not priced and the 3-term Chebyshev polynomial kernel with two state variables is still preferred.⁹

3.4 Discussion

Our robustness test indicates that the 3-term polynomial approximation with the real interest rate and the maximum Sharpe ratio emerges as our preferred pricing kernel. This contrasts with the full sample results in which all three state variables, including implied volatility, are priced. The difference in the priced state variables

⁹We thank an anonymous referee for suggesting robustness testing. Detailed results are available from the authors upon request.

is likely due to the fact that the LIBOR rate has been unusually stable due to the success of implicit or explicit inflation targeting over the course of the Great Moderation up to the onset of the banking crisis in mid-2007. Both the UK and US economies enjoyed unusually stable macroeconomic and monetary policy regimes over the period up to 2007 and LIBOR options traded against this background. It has been argued that the low interest rate policy during the latter part of the Great Moderation may have contributed to the onset of the banking and financial crisis in 2007 as this policy focused on goods price inflation and ignored incipient asset price inflation. This may explain the lack of significance of implied LIBOR volatility as a state variable in the option pricing kernel. *More specifically, this may be because the underlying asset, the LIBOR 6-month interest rate, was less volatile before the financial crisis. We find that the variance of the LIBOR 6-month interest rate over the entire sample period is 0.71 while it is 0.61 in the pre-crisis period. Therefore, the volatility becomes an important state variable with the onset of the banking crisis for options on interest rate futures.*

4 Concluding remarks

In this paper, we evaluate option pricing models for LIBOR interest rate futures within the intertemporal CAPM framework employing the second HJD criterion and monthly return portfolios on these options from January 2000 to February 2008. Using the Bayesian methodology of Wang and Zhang (2012), the empirical results show that the three-term polynomial pricing kernel with three state variables

- the real interest rate, maximum Sharpe ratio and implied option volatility - has the smallest second HJD and thus is the preferred pricing kernel. The latter is strictly positive and everywhere monotonically decreasing in market returns in conformity with economic theory. However, when we confine our sample to the pre-crisis period up to December 2006, the three-term polynomial pricing kernel with just two state variables is preferred and implied option volatility is not priced. The success of monetary policy during the Great Moderation in maintaining low and stable interest rates from the early 1990s seems the most plausible explanation for this.

In this paper, we have applied a Bayesian-based methodology in using the second HJD to choose our preferred pricing kernel. In future work, it would be interesting to apply the recently proposed approach of Li, Xu, and Zhang (2010) which adopts conventional econometric methods in estimating the second HJD.

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Table 1. Summary statistics of the LIBOR option portfolio returns

moneyiness	no.	mean	std	skew	kurt	min	max	JB test
Call Options								
≤ 0	51	0.099	0.687	2.183	7.051	-0.724	3.576	252.388 (0.000)
≤ 0.01	48	0.121	0.425	2.046	7.134	-0.476	2.348	247.620 (0.000)
≤ 0.02	48	0.049	0.169	1.034	2.400	-0.295	0.761	36.606 (0.000)
≤ 0.03	42	0.028	0.102	0.581	1.129	-0.210	0.404	9.417 (0.000)
> 0.03	45	0.019	0.066	0.335	0.288	-0.124	0.226	1.943 (0.378)
Put Options								
≤ -0.01	28	-0.077	0.658	2.849	11.067	-0.715	3.631	567.992 (0.000)
≤ 0	32	-0.052	0.664	2.356	6.368	-0.733	2.897	231.699 (0.000)
≤ 0.01	49	0.026	0.400	1.960	4.878	-0.465	1.841	144.581 (0.000)
≤ 0.02	50	-0.004	0.161	1.430	3.666	-0.301	0.702	79.260 (0.000)
> 0.02	59	-0.006	0.096	0.752	1.939	-0.216	0.347	21.649 (0.000)

This table provides summary statistics of the monthly portfolio returns with LIBOR futures options from January 2000 to February 2008. The p -values for the Jarque-Bera test of normality are reported in the parentheses.

Table 2. Summary statistics of UK government bond yield and state variables

Panel A: UK government bond yields (%)									
Maturity (yr)	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	10.0
Mean	4.74	4.84	4.93	4.99	5.02	5.05	5.07	5.08	5.10
Stdev	1.56	1.41	1.29	1.21	1.16	1.12	1.10	1.08	1.07
Max	7.23	7.03	7.13	7.38	7.58	7.75	7.89	8.00	8.18
Min	0.59	1.21	1.74	2.17	2.52	2.81	2.99	3.15	3.48

Panel B: State variables								
	r	Δr	η	$\Delta \eta$	σ	$\Delta \sigma$	r_f	r_W
Mean	0.025	0.002	0.838	0.003	0.153	0.001	0.052	0.050
Stdev	0.075	0.027	0.949	0.289	0.050	0.016	0.011	0.476
Skew	-0.323	3.258	0.043	-3.069	0.510	1.279	0.244	-0.283

Panel A provides summary statistics of UK government bond yields of different maturities from January 1996 to February 2008. These data are taken from the Bank of England website. Panel B shows summary statistics of the state variables and the innovations of the state variables used in the pricing kernels. The state variables are real interest rate r , inflation π , maximum Sharpe ratio η , riskfree rate r , and returns on aggregate wealth proxied by FTSE-100 index returns.

Table 3. GMM parameter estimates of the pricing kernels

β	γ	r	η	σ	J_T
Panel A: Iso-elastic power function					
0.729	1.169	14.195	-0.583	12.938	145.832
(0.000)	(0.649)	(0.000)	(0.037)	(0.010)	(0.000)
0.629	2.481	17.170	-0.559		126.184
(0.000)	(0.285)	(0.000)	(0.034)		(0.000)
0.627	3.666	12.881			123.973
(0.000)	(0.088)	(0.000)			(0.000)
1.002	1.868				152.801
(0.000)	(0.000)				(0.000)
Panel B: Polynomial approximation (n=3)					
		41.828	-1.448	17.665	26.369
		(0.000)	(0.000)	(0.043)	(0.999)
		26.151	-1.481		47.159
		(0.000)	(0.040)		(0.734)
		18.465			53.044
		(0.000)			(0.550)
Panel C: Polynomial approximation (n=4)					
		41.847	-1.598	7.942	73.750
		(0.000)	(0.000)	(0.330)	(0.025)
		33.640	-1.473		36.878
		(0.000)	(0.000)		(0.955)
		19.795			48.619
		(0.000)			(0.681)

Panel A gives the parameters estimated for the iso-elastic power function

$$m_{t+1} = \beta(R_{w,t+1})^{-\gamma} \exp^{b_1 \Delta r_{t+1} + b_2 \Delta \eta_{t+1} + b_3 \Delta \sigma_{t+1}} .$$

Panels B and C present the parameters estimated for the polynomial pricing kernel

$$m_{t+1} = \wp^n(R_{w,t+1}) \exp^{b_1 \Delta r_{t+1} + b_2 \Delta \eta_{t+1} + b_3 \Delta \sigma_{t+1}} ,$$

where γ is the risk aversion parameter, $R_{w,t+1}$ is portfolio returns, $\wp^n(R_{w,t+1})$ consists of n-term Chebyshev polynomials, and r , η , and σ stand for the real interest rate, maximum Sharpe ratio, and volatility, respectively. J_T is over-identifying test statistic. The p -values are reported in the parentheses.

Table 4. Second HJ distance parameter estimates of the pricing kernels

β	γ	r	η	σ	2nd HJ distance (mean)	posterior mean $Dist_{HJ2}^A/Dist_{HJ2}^B$
Panel A: Iso-elastic power function						
0.585	4.607	21.028	-1.053	11.356	0.352	4.241
0.655	4.574	19.918	-0.747		0.479	5.771
0.768	2.778	15.042			0.392	4.723
0.988	-3.646				0.526	6.337
Panel B: Polynomial approximation (n=3)						
		18.218	-1.314	12.756	0.083	1.000
		16.912	-0.995		0.121	1.458
		13.117			0.204	2.458
Panel C: Polynomial approximation (n=4)						
		19.522	-1.558	9.411	0.314	3.483
		16.559	-0.852		0.310	3.735
		11.603			0.248	2.988

Panel A gives the parameters estimated for the iso-elastic power function

$$m_{t+1} = \beta(R_{w,t+1})^{-\gamma} \exp^{b_1 \Delta r_{t+1} + b_2 \Delta \eta_{t+1} + b_3 \Delta \sigma_{t+1}}.$$

Panels B and C present the parameters estimated for the polynomial pricing kernel

$$m_{t+1} = \wp^n(R_{w,t+1}) \exp^{b_1 \Delta r_{t+1} + b_2 \Delta \eta_{t+1} + b_3 \Delta \sigma_{t+1}},$$

where γ is the risk aversion parameter, $R_{w,t+1}$ is portfolio returns, $\wp^n(R_{w,t+1})$ consists of n-term Chebyshev polynomials, and r , η , and σ stand for the real interest rate, maximum Sharpe ratio, and volatility, respectively. The posterior mean of the ratio of the 2nd HJ distance are also reported, in which the 2nd HJ distance of the 3-term polynomial approximation with three state variables are employed as the denominator $Dist_{HJ2}^B$.

Table 5. GMM parameter estimates of the pricing kernels (Pre-crisis samples)

β	γ	r	η	σ	J_T
Panel A: Iso-elastic power function					
0.533	5.211	44.834	-3.354	10.810	153.960
(0)	(0.120)	(0)	(0)	(0.187)	(0)
0.642	5.869	39.694	-3.039		147.641
(0)	(0.012)	(0)	(0)		(0)
0.901	3.360	11.431			158.935
(0)	(0.034)	(0)			(0)
1.001	4.116				153.137
(0)	(0)				(0)
Panel B: Polynomial approximation (n=3)					
		45.960	-3.481	14.949	48.859
		(0.002)	(0.001)	(0.248)	(0.636)
		47.773	-3.079		39.632
		(0.020)	(0.002)		(0.928)
		36.667			52.468
		(0.002)			(0.572)
Panel C: Polynomial approximation (n=4)					
		44.935	-3.602	12.966	39.383
		(0.029)	(0.010)	(0.346)	(0.901)
		40.945	-3.695		36.888
		(0.025)	(0.005)		(0.955)
		20.794			42.555
		(0.012)			(0.870)

Panel A gives the parameters estimated for the iso-elastic power function

$$m_{t+1} = \beta(R_{w,t+1})^{-\gamma} \exp^{b_1 \Delta r_{t+1} + b_2 \Delta \eta_{t+1} + b_3 \Delta \sigma_{t+1}} .$$

Panels B and C present the parameters estimated for the polynomial pricing kernel

$$m_{t+1} = \wp^n(R_{w,t+1}) \exp^{b_1 \Delta r_{t+1} + b_2 \Delta \eta_{t+1} + b_3 \Delta \sigma_{t+1}} ,$$

where γ is the risk aversion parameter, $R_{w,t+1}$ is portfolio returns, $\wp^n(R_{w,t+1})$ consists of n-term Chebyshev polynomials, and r , η , and σ stand for the real interest rate, maximum Sharpe ratio, and volatility, respectively. J_T is over-identifying test statistic. The p -values are reported in the parentheses.

Table 6. Second HJ distance parameter estimates of the pricing kernels (Pre-crisis samples)

β	γ	r	η	σ	2nd HJ distance (mean)	posterior mean $Dist_{HJ2}^A/Dist_{HJ2}^B$
Panel A: Iso-elastic power function						
0.915	4.050	39.379	-2.967	10.034	0.313	2.285
0.755	4.356	33.224	-3.331		0.370	2.701
0.811	4.306	10.970			0.252	1.839
0.969	4.162				0.289	2.109
Panel B: Polynomial approximation (n=3)						
		40.219	-3.397	12.468	0.212	1.547
		41.597	-4.004		0.137	1.000
		18.143			0.178	1.299
Panel C: Polynomial approximation (n=4)						
		42.954	-3.083	15.170	0.244	1.781
		46.828	-3.270		0.294	2.146
		26.982			0.201	1.467

Panel A gives the parameters estimated for the iso-elastic power function

$$m_{t+1} = \beta(R_{w,t+1})^{-\gamma} \exp^{b_1 \Delta r_{t+1} + b_2 \Delta \eta_{t+1} + b_3 \Delta \sigma_{t+1}} .$$

Panels B and C present the parameters estimated for the polynomial pricing kernel

$$m_{t+1} = \wp^n(R_{w,t+1}) \exp^{b_1 \Delta r_{t+1} + b_2 \Delta \eta_{t+1} + b_3 \Delta \sigma_{t+1}} ,$$

where γ is the risk aversion parameter, $R_{w,t+1}$ is portfolio returns, $\wp^n(R_{w,t+1})$ consists of n-term Chebyshev polynomials, and r , η , and σ stand for the real interest rate, maximum Sharpe ratio, and volatility, respectively. The posterior mean of the ratio of the 2nd HJ distance are also reported, in which the 2nd HJ distance of the 3-term polynomial approximation with two state variables are employed as the denominator $Dist_{HJ2}^B$.

Figure 1. Market-related component of the pricing kernels estimated by the GMM

The pricing kernels are either with real interest rate (1 sv), or with real interest rate and maximum Sharpe ratio (2 sv), or with real interest rate, maximum Sharpe ratio, and volatility (3 sv), where sv stands for state variables.

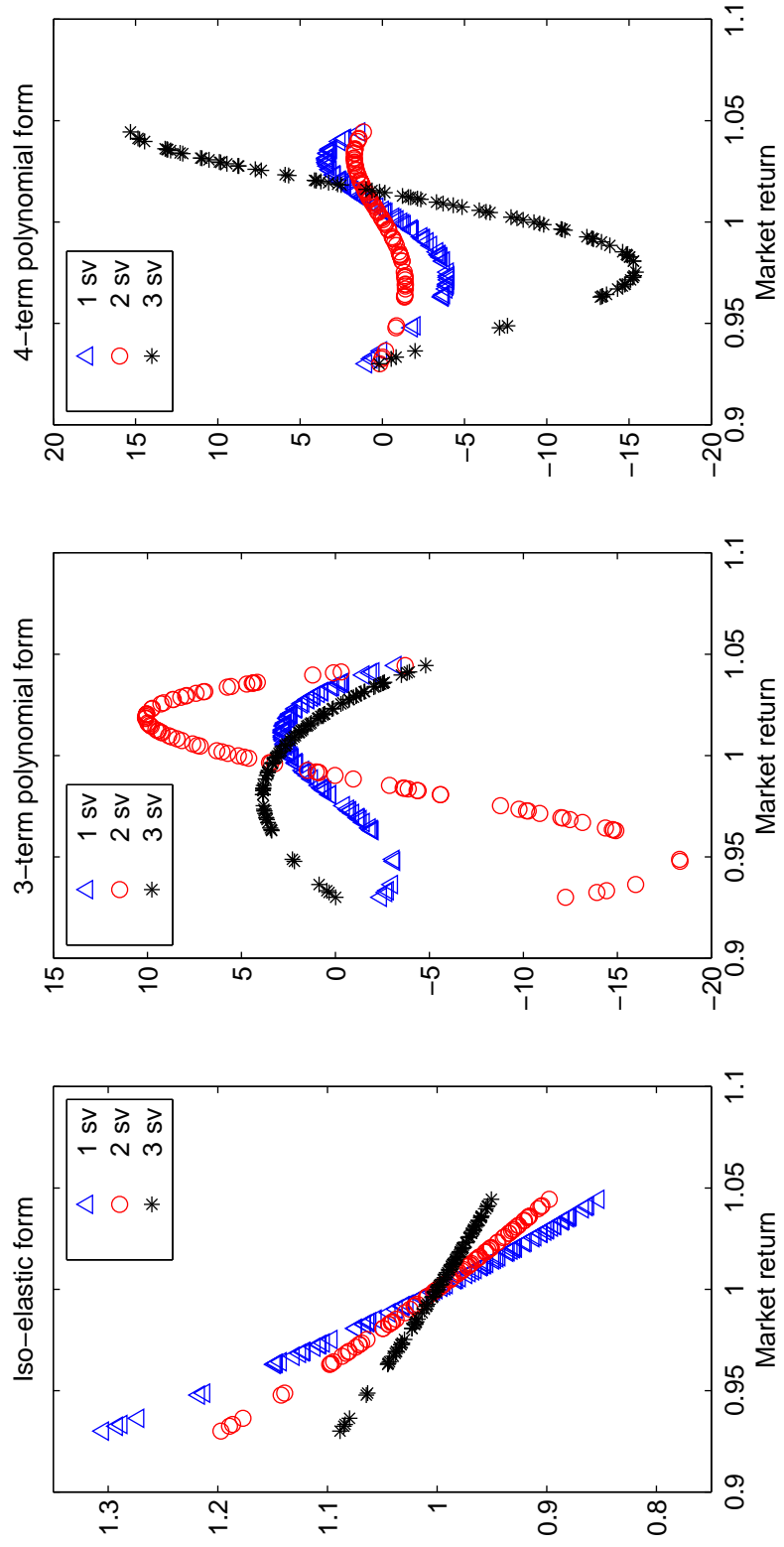


Figure 2. Market-related component of the pricing kernels estimated by minimizing the second HJ distance

The pricing kernels are either with real interest rate (1 sv), with real interest rate and maximum Sharpe ratio (2 sv), or with real interest rate, maximum Sharpe ratio, and volatility (3 sv), where sv stands for state variables.

